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Undergraduates' Error Patterns And Misconceptions In Further Differential Equations

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Abstract

A great concern about the high failure rate among the civil engineering undergraduates in the mathematics course i.e. Further Differential Equations arises recently in Universiti Teknologi MARA Campus Penang. The Further Differential Equations is one of the important courses in the engineering program. This paper describes a study undertaken to investigate the error patterns and misconceptions in the Further Differential Equations demonstrated by the civil engineering students. The study is based on the tests and final examination which were taken by a group of forty civil engineering students. Suggestions are proposed for the lecturers and the students to rectify the common error patterns and misconceptions.

Introduction

In mathematics education, numerous studies about the errors and misconceptions in mathematics learning had been carried out by researchers (Siti Aishah & Noor 'Aina, 2005; Luneta & Makonye, 2010; Sofronas, DeFranco, Vinsonhaler, Gorgievshi, Schroeder, & Hamelin, 2011; Muzangwa & Chifamba, 2012; Sarwadi & Shahrill, 2014; Maisurah, Siti Balqis, Norazah, & Fadzilawani Astifar, 2015). According to Radatz (1980), students' errors are causally determined, very often systematic and persistent in their several years of schooling. Once the students have failed to grasp the fundamental knowledge in their early learning, they may have develop misconceptions and thereby produce wrong procedures solving a problem. In addition, Radatz (1979) categorized the common errors made by students as (1) semantic difference between mathematics language and natural language where the students need to transfer the real life problem by understanding and using mathematical concepts, symbols and equations; (2) difficulties in processing iconic and visual representation of mathematical knowledge; (3) deficiency in requisite skills, facts, and concepts; for instance, learners may not able to memorize related information in solving problems; (4) incorrect associations or rigidity; and (5) application of irrelevant rules or strategies.

Li (2006) indicated that students' errors are the results of misconception. Among many different types of errors, the systematic errors occur to many students over a long time period. The cause of systematic errors may relate to student's procedure knowledge, conceptual knowledge, or links between these two types of knowledge. Generally misconceptions manifest through errors. An error can be a mistake, blunder, miscalculation or misjudge and such category falls under unsystematic errors (Muzangwa & Chifamba, 2012). They also reported some errors and misconceptions committed by students in calculus due to lack of basic knowledge in algebra and mathematical thinking skill.

Further Differential Equations is one of the courses all the undergraduate civil engineering students must take in Universiti Teknologi MARA Campus Penang. The engineering students are compulsory to pass two mathematics courses namely Calculus for Engineers and Further Calculus for Engineers before they undertake the Further Differential Equations. Sazhin (1998) perceived mathematics as a language to express physical, chemical and engineering laws. In order to motivate a thorough approach to students' learning, all general equations should be illustrated by practical numerical examples. Therefore, knowledge of mathematics covers algebra, differential equations and partial differential equations which are essential for all engineering students. A good engineer should possess strong mathematics skills since the application of engineering require good understanding on the mathematical concept.

Recently, the high failure rate in Further Differential Equations had gained the major attention and concern in the Faculty of Civil Engineering in Universiti Teknologi MARA Campus Penang. The aim of this study is to investigate error patterns and misconceptions among the civil engineering students who took the Further Differential Equations in the university. The findings of this study could assist the lecturers and students to rectify the errors in their teaching and learning in Further Differential Equations.

METHODOLOGY

In this study, error patterns and misconceptions in the Further Differential Equations offered to civil engineering undergraduates at University Teknologi MARA Campus Penang are investigated. Tests and final examination scripts of a group of forty civil engineering students were studied. The assessments cover all the main topics of Further Differential Equations, i.e. Series Solution for Second Order Linear Differential Equations with Variable Coefficient, Legendre Polynomials, Fourier Series and Partial Differentiation Equations. Table 1 shows the mathematical concepts or skills that are embedded into the four main topics in the Further Differential Equations.

TABLE 1
THE MATHEMATICAL CONCEPTS OR SKILLS IN FURTHER DIFFERENTIAL EQUATIONS

Topics	**Mathematical Concepts or Skills**
Series Solution for Second Order Linear Differential Equations with Variable Coefficient	Prior knowledge in power series and algebra manipulation.
Legendre Polynomials	Techniques of differentiation and integration.
Fourier Series	Prior knowledge in graph sketching and trigonometric functions. Apply the correct techniques of integration.
Partial Differentiation Equations	Prior knowledge in solving the first order and second order differential equations.

RESULTS AND DISCUSSION

Based on the analysis from the tests and final examination sample scripts, error patterns and misconceptions of three topics are identified.

Error patterns in the topic of Series Solution for Second Order Linear Differential Equations

The students always have the high tendency to make errors in solving the problems of Series Solution for Second Order Linear Differential Equations with Variable Coefficient. The sample question and selected students' worked solutions (student A and B) are illustrated in the following figure (Fig. 1).

Question 1: Find the solution of the differential equation $\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$ at the ordinary point x = 1 up to the terms $(x-1)^3$.	
Student A:	**Student B**:
Let $t = x - 1$ $\frac{d^2y}{dt^2} - t\frac{dy}{dt} - y = 0$	Let $t = x - 1$ $x = t - 1$ $\frac{d^2y}{dt^2} - t - 1\frac{dy}{dt} - y = 0$

Fig. 1.
Worked solution showing different error patterns in the series solution for second order differential equations

In order to solve the differential equation in above figure (Fig. 1), the assumption of $t = x - 1$ should be made. Then, the differential equation should be transformed to $\frac{d^2y}{dt^2} - (t+1)\frac{dy}{dt} - y = 0$.

In Fig. 1, it shows that student A applied the correct assumption that $t = x - 1$ is an ordinary point solution of the differential equation. Later, the student A just simply substituted the variable *x* with *t* in the given second order differential equation. The student B also showed the carelessness in algebra when rewriting *x* as the subject.

The student B directly substituted $x = t - 1$ into the given differential equation by neglecting the law of algebra (omitting the bracket for the expression $t - 1$). These errors that occurred frequently in the primarily stage will cause the students unable to obtain the actual answer.

Error patterns in the topic of Fourier Series

Three types of error patterns are usually found in the topic of Fourier series i.e. (i) The students can't sketch the graph correctly, (ii) they have problem of transferring the graphical information into an appropriate function, and (iii) they do not apply the correct method of integration. The sample question and the different selected students' worked solutions (student C, D and E) are illustrated in the following figure (Fig. 2).

To answer the question 2a), the worked solution by student C in Figure 2 shows that the student sketched the wrong graph of function f(x) = 2 in the interval between 0 and 2π. The correct graph of f(x) = 2 is supposed to be a horizontal line. Thus, the student misinterprets the graph of f(x) as an odd function. The student will get partial marks in the graph, but he failed to get any marks in the following steps. The student C failed to define the correct function because Fourier Series require students to represent the given function with a graph.

The worked solution by student D in Fig. 2 depicts the student used the wrong formula to solve the coefficient, a_o, in the question 2b). The student was not able to distinguish between the formula of general Fourier series and Fourier Cosine or Fourier Sine series in solving coefficient, a_o. This shows that the student did not possess a thorough understanding in even and odd functions.

Moreover, in the worked solution by student E as shown in Fig. 2, although the student knew how to use tabular method to perform integration by parts in solving the coefficient of Fourier series, but the student could not understand the function by keeping the constant and integrate the trigonometric functions properly. Hence, the students who made these errors are perceived lack of the calculus knowledge.

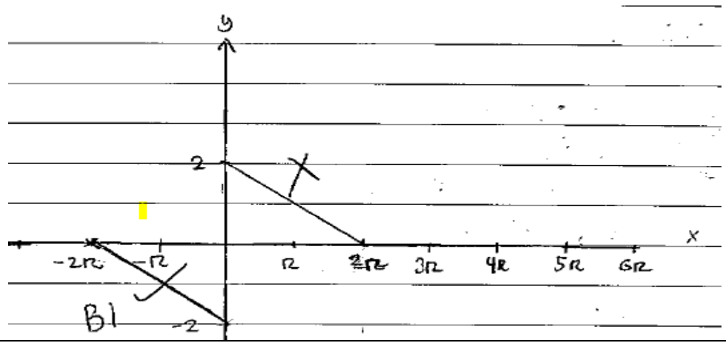
Question 2:

A periodic function with $f(x) = f(x + 2\pi)$ is defined as $f(x) = \begin{cases} -\dfrac{x}{\pi} - 2, & -2\pi < x < 0 \\ 2, & 0 < x < 2\pi \end{cases}$.

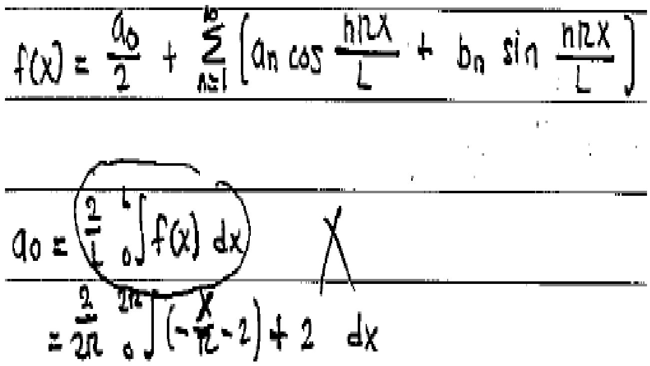
a) Sketch the graph of f(x).

b) Find a Fourier series expansion of f(x).

Student C:



Student D:



Student E:

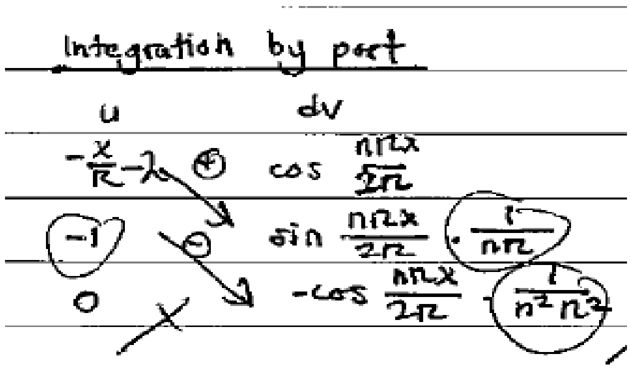


Fig. 2.
Worked solution showing different error patterns in the Fourier Series

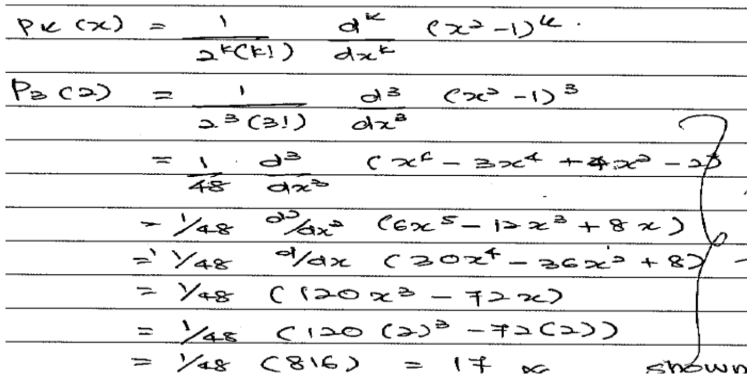
Error patterns in the topic of Legendre Polynomials

In order to solve the Legendre polynomials in Fig. 3, the students need to know how to use the generating function to prove Legendre polynomials. The students were in a dilemma of using different definitions and methods in proving Legendre Polynomials. Thus, they attempted to answer the question by guessing one of the definitions and methods. It is clearly shown in the worked solutions by student F and student G in Fig. 3. No marks were given in the solutions with incorrect associations of definitions and methods. The student H in Fig. 3 initially applied the correct definition to prove the Legendre Polynomial. However, the student applied the wrong strategies in the later stage. Hence, the solution was incorrect. This shows that the students are lack of critical thinking skill. Therefore, lecturers should carry out small group discussions by giving a variety of critical thinking problems. Students will learn how to use the correct method deriving the Legendre Polynomials and the definition in solving the problems involved functions.

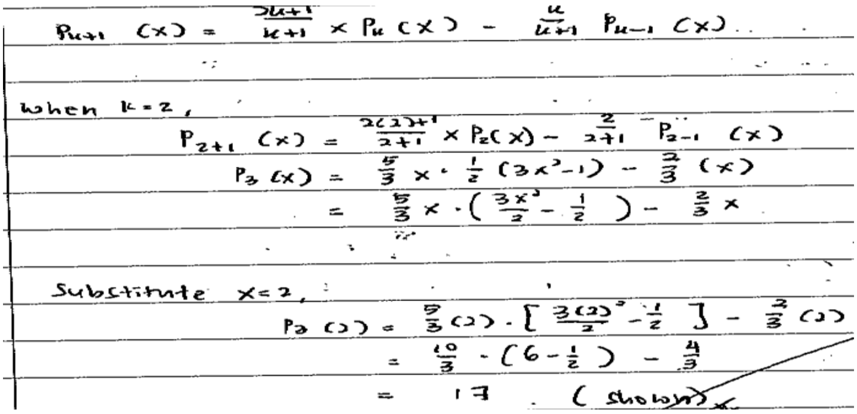
Question 3:

Use the generating function for Legendre Polynomial to show that $P_3(2) = 17$.

Student F:



Student G:



Student H:

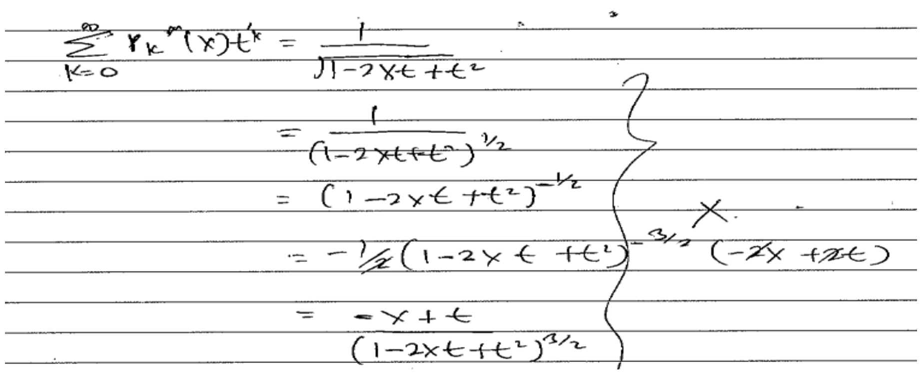


Fig. 3.
Worked solution showing different error patterns in the Legendre Polynomials

CONCLUSION

Further Differential Equations is an important course for the civil engineering undergraduates. In this study, we pinpointed some error patterns and misconceptions made by undergraduates who took Further Differential Equations. The discussion in this paper indicated that the students are still weak in their basic algebra, calculus and functions. Furthermore, the study highlighted potential error patterns and misconceptions in solving questions in Further Differential Equations. This could benefit both lecturers and students in their teaching and learning in the future.

Based on the discussion and the conclusion, the following suggestions were proposed:

(1) Necessary steps in the manipulation of algebraic equations should be highlighted in the formal lecture and tutorial. For example, lecturers need to review some important basic laws of algebra before delivering the relevant topic.

(2) Students should devote more time to learning of calculus because they will apply calculus in learning all the topics in Further Differential Equations. The strong foundation in calculus will facilitate their learning in the Further Differential Equations.

Adequate instructional materials especially handouts should be provided to the students. Element of small group teaching and formative assessment should be implemented in the teaching and learning Further Differential Equations.

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